Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Nawala - Nugegoda

Course: MPZ 3231-Engineering Mathematics IA **Model Answer No·03**

Academic Year – 2013/2014

1. a)

$$y = f(x) = \frac{x^2 - x}{x+1}$$

$$\frac{dy}{dx} = f'(x) = \frac{(x+1)(2x-1) - (x^2 - x)}{(x+1)^2} = \frac{(x^2 + 2x - 1)}{(x+1)^2}$$

$$f'(x) = \frac{(x+1)^2 - 2}{(x+1)^2} = \frac{\left(x+1+\sqrt{2}\right)\left(x+1-\sqrt{2}\right)}{(x+1)^2}$$

$$\frac{dy}{dx} = f'(x) = \frac{\left[x - \left(-1-\sqrt{2}\right)\right]\left[x - \left(-1+\sqrt{2}\right)\right]}{(x+1)^2} \qquad x \neq -1$$

$x < -\left(1+\sqrt{2}\right)$ $f'(x) > 0$

$x = -\left(1+\sqrt{2}\right)$ $f'(x) = 0$

When $x = -\left(1+\sqrt{2}\right)$ $f(x)$ has a local maximum point

$$\left[-\left(1+\sqrt{2}\right), -\left(1+\sqrt{2}\right)^2\right]$$

$-\left(1+\sqrt{2}\right) < x < \left(-1+\sqrt{2}\right)$ $f'(x) < 0$

$x = \left(-1+\sqrt{2}\right)$ $f'(x) = 0$

$\left(-1+\sqrt{2}\right) < x$ $f'(x) > 0$

When $x = \sqrt{2} - 1$ $f(x)$ has a local minimum point

$$\left[\left(\sqrt{2}-1\right), \left(\sqrt{2}-1\right)^2\right]$$

$x \to +\infty \qquad y \to +\infty$

$x \to -\infty \qquad y \to -\infty$

When $x = -1$ y is not defined.

$\therefore$ $x = -1$ is the vertical asymptotes of the curve

When $x = 0$ and $x - 1$, $y = 0$

Let assume that slant asymptote $y = ax + b$

$$x \to +\infty \ \ \{f(x) - ax - b\} = 0$$

$$x \to \infty \ \ x\left\{\frac{f(x)}{x} - a - \frac{b}{x}\right\} = 0$$

$$x \to \infty \quad \left\{\frac{f(x)}{x} - a - \frac{b}{x}\right\} = 0$$

$$x \to \infty \quad \frac{f(x)}{x} = a$$

$$x \to \infty \quad \frac{x^2 - x/_{x+1}}{x} = a$$

$$x \to \infty \quad \frac{x^2 - x}{x^2 + x} = a$$

$$x \to \infty \quad \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = 1 = a$$

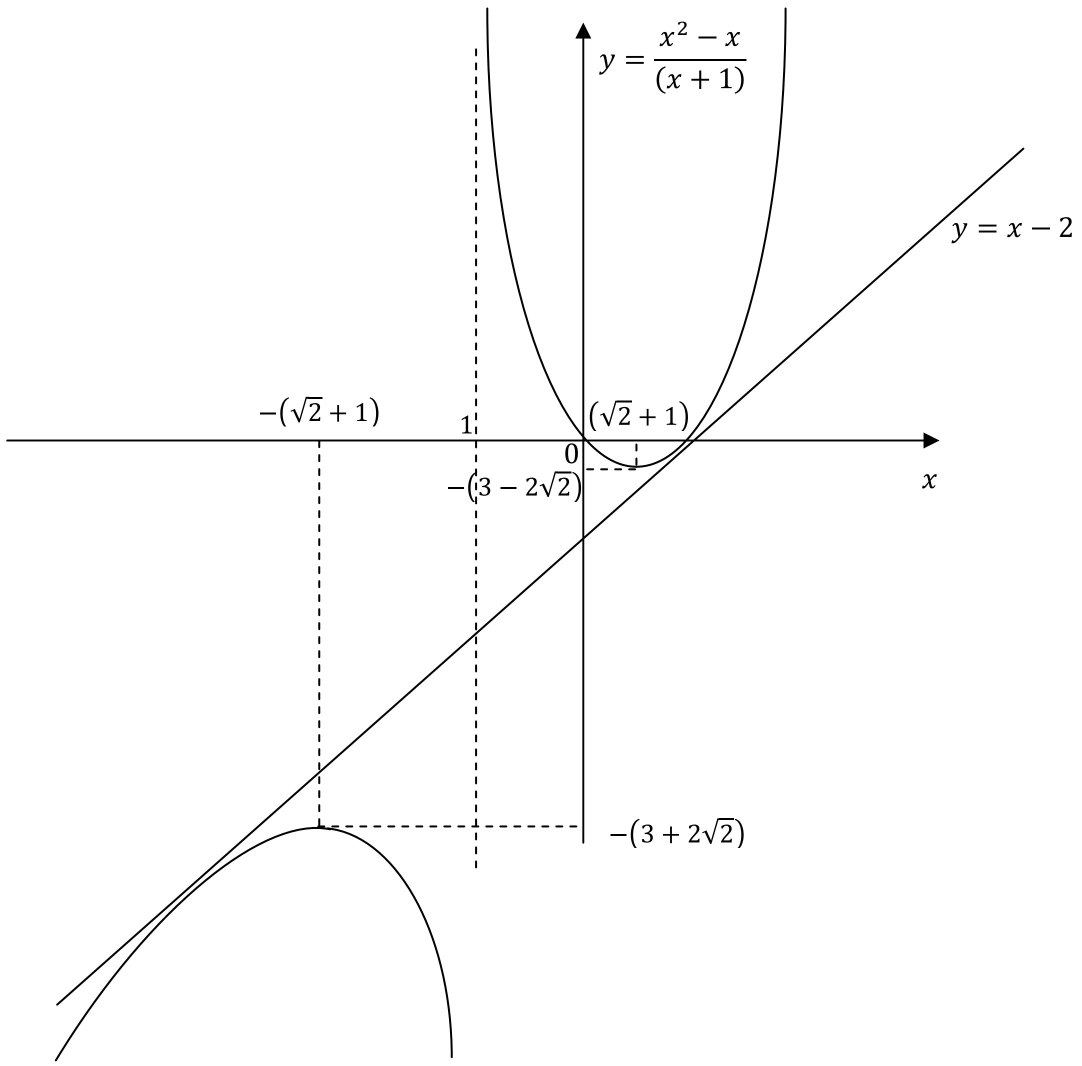
$$\therefore \ lim_{x\to\infty} \quad \frac{x^2 - x/_{x+1}}{x} = 1 = a$$

$$b = x \to \infty \quad [f(x) - ax] = x \to \infty \quad \left[\frac{x^2 - x}{x+1} - x\right]$$

$$= \quad x \to \infty \quad \left[\frac{x^2 - x - x^2 - x}{(x+1)}\right]$$

$$= \quad x \to \infty \quad \left[\frac{-2x}{(x+1)}\right] = -2$$

$\therefore$ The equation of the slant asymptote $y = x - 2$



b)

$$y^2 = \frac{x}{x-2} \quad ; y^2 \geq 0 \quad x(x-2) \geq 0$$

$\therefore \quad x \leq 0$ and $x > 2$

$\therefore$ curve is symmetric about the x axis

$$y = \pm\sqrt{\frac{x}{x-2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{x-2}\right]^{-\frac{1}{2}} \frac{[(x-2)\times 1 - x]}{(x-2)^2} = -\sqrt{\frac{x-2}{x}}\frac{1}{(x-2)^2}$$

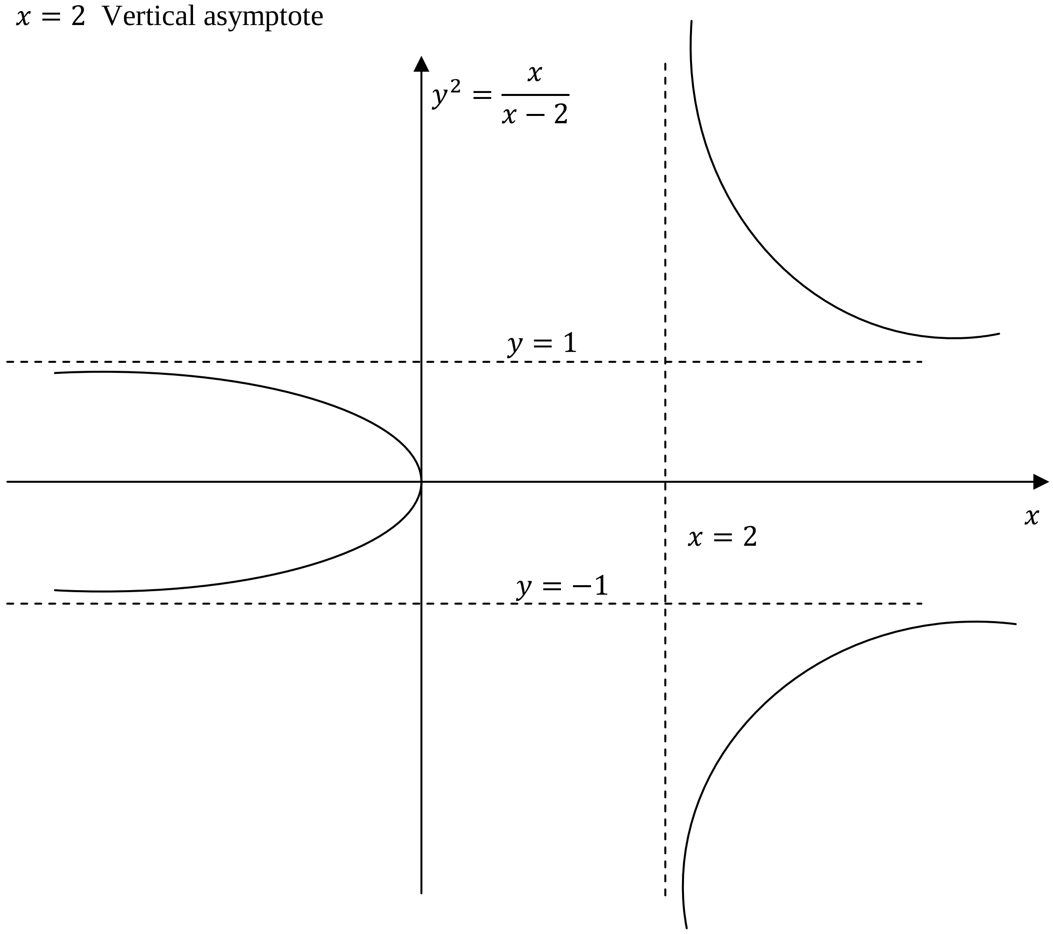
$$= \frac{-1}{\sqrt{x(x-2)^3}}$$

$\therefore$ when $x = 0 \quad \frac{dy}{dx}$ not defined

$0 \leq x \qquad \frac{dy}{dx} < 0 \qquad x > 2 \qquad \frac{dy}{dx} < 0$

$x \to \infty \;\; y = \pm 1$ (horizontal asymptotes)

$x = 2$ Vertical asymptote



c)

$$y = f(x) = \frac{x^2 - x - 12}{x + 2} = \frac{(x-4)(x+3)}{(x+2)}$$

$$\frac{dy}{dx} = f'(x) = \frac{(x+2)(2x-1) - (x^2 - x - 12)}{(x+2)^2} = \frac{x^2 + 4x + 10}{(x+2)^2}$$

$\therefore \quad f'(x), \; \frac{dy}{dx} > 0$ for all values of x

$x + 2 = 0$ is vertical asymptote

$x \to \infty \qquad y \to \infty$

$x \to -\infty \quad y \to -\infty$

To find slant asymptotes,

Let assume that slant asymptote $y = ax + b$

$\therefore x \to +\infty \;\; \{f(x) - ax - b\} = 0$

$$x \to \infty \quad x\left\{\frac{f(x)}{x} - a - \frac{b}{x}\right\} = 0$$

$$x \to \infty \quad \left\{\frac{f(x)}{x} - a - \frac{b}{x}\right\} = 0$$

$$x \to \infty \quad \frac{f(x)}{x} = a$$

$$\frac{f(x)}{x} = \frac{x^2 - x - 12}{x(x+2)} \qquad \therefore\ lim_{x\to\infty} \quad \frac{f(x)}{x} = 1 = a$$

$$b = x \to \infty \quad [f(x) - ax] = x \to \infty \quad \left[\frac{x^2 - x - 12}{x+2} - x\right]$$

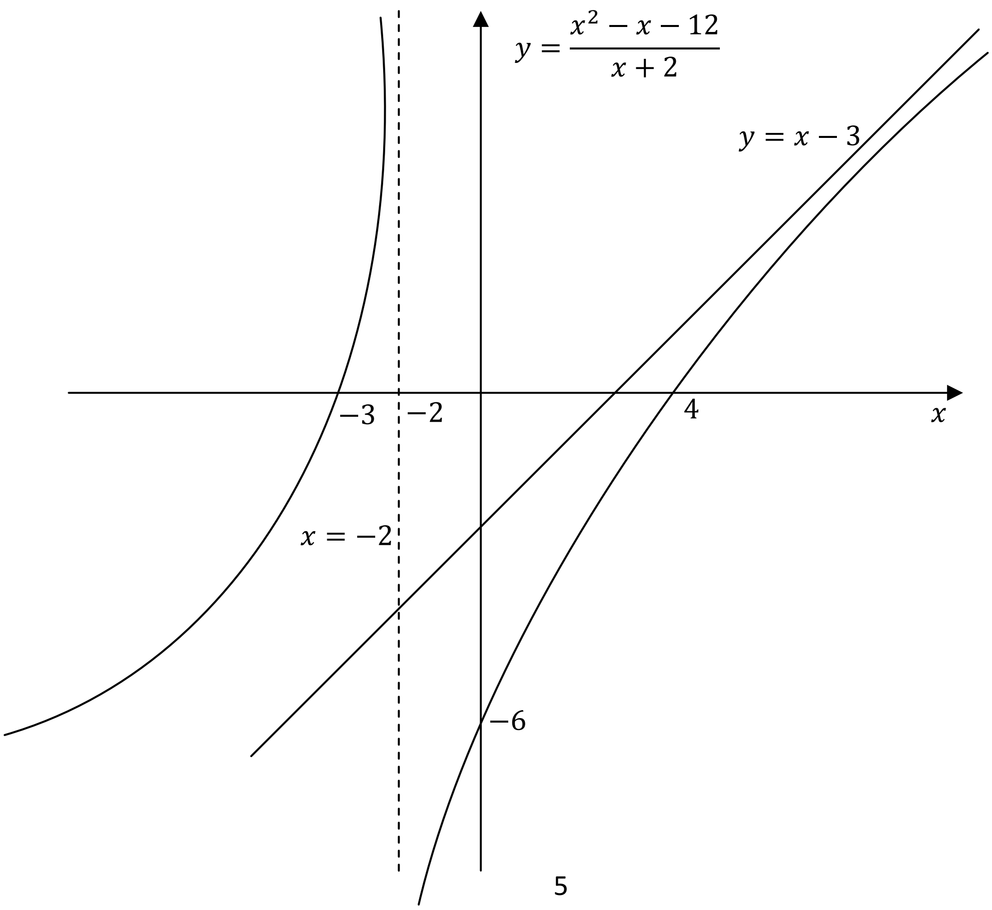
$$= \quad x \to \infty \quad \left[\frac{-x - 12 - 2x}{(x+2)} = -3\right]$$

$\therefore$ The equation of the slant asymptote $y = x - 3$

$$y = f(x) = \frac{x^2 - x - 12}{x+2} = \frac{(x-4)(x+3)}{(x+2)}$$

$\therefore \quad y = 0$ when $x = 4$ and $x = -3$

$x = 0 \qquad y = -6$



2. a)

$$lim_{x\to4}\ \frac{x^2-16}{x^2+x-20}=lim_{x\to4}\ \frac{(x-4)(x+4)}{(x-4)(x+5)}=lim_{x\to4}\ \frac{(x+4)}{(x+5)}=\frac{8}{9}$$

b)

$$lim_{x\to1}\ \frac{x^5-2x^3-4x^2+9x-4}{x^4-2x^3+2x-1}$$

$$lim_{x\to1}\ \frac{(x-1)(x^4+x^3-x^2-5x+4)}{(x-1)(x^3-x^2-x+1)}$$

$$lim_{x\to1}\ \frac{(x^4+x^3-x^2-5x+4)}{(x^3-x^2-x+1)}$$

$$lim_{x\to1}\ \frac{(x-1)(x^3+2x^2+x-4)}{(x-1)(x^2-1)}$$

$$lim_{x\to1}\ \frac{(x^3+2x^2+x-4)}{(x^2-1)}$$

$$lim_{x\to1}\ \frac{(x-1)(x^2+3x+4)}{(x-1)(x+1)}$$

$$lim_{x\to1}\ \frac{(x^2+3x+4)}{(x+1)}=\frac{8}{2}=4$$

c)

$$lim_{x\to0}\ \left(\frac{\sin x-x}{x^3}\right)=lim_{x\to0}\left(\frac{\cos x-1}{3x^3}\right)$$ L' hospital Rule

$$=lim_{x\to0}\left(\frac{2\sin^2\frac{x}{2}}{3x^3}\right)$$

$$=lim_{x\to0}\frac{2}{3.4}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$=\frac{1}{6}$$

d).

$$lim_{x\to 0}\ \frac{e^{ax}-e^{-ax}}{\log_e[1+bx]}$$

By using L Hospital Rule

$$lim_{x\to 0}\ \frac{f(x)}{g(x)} = lim_{x\to 0}\ \frac{f'(x)}{g'(x)}$$

$$= lim_{x\to 0}\ \frac{e^{ax}-e^{-ax}}{\log_e[1+bx]} = \frac{lim_{x\to 0} a(e^{ax}+e^{-ax})}{lim_{x\to 0}\ \left[\frac{b}{1+bx}\right]}$$

$$= \frac{2a}{b}$$

e).

$$lim_{x\to 0}\frac{e^{x}-e^{-x}-2x}{x-\sin x}$$

$$= lim_{x\to 0}\frac{e^{x}-e^{-x}-2}{1-\cos x}$$

$$= lim_{x\to 0}\frac{e^{x}-e^{-x}}{\sin x}$$

$$= lim_{x\to 0}\frac{e^{-x}(e^{2x}-1)}{\sin x}$$

$$= lim_{x\to 0}\frac{e^{-x}(e^{2x}-1)}{2x}\left[\frac{2}{\frac{\sin x}{x}}\right] \ = 2$$

3. a)

$z = f(x+ay) + f(x-ay)$

$$\frac{\partial z}{\partial y} = f'(x+ay).a + f'(x-ay).(-a)$$

$$\frac{\partial^2 z}{\partial y^2} = a^2[f'(x+ay) + f'(x-ay).] --- (1)$$

$$\frac{\partial z}{\partial x} = f'(x+ay) + f'(x-ay)$$

$$\frac{\partial^2 z}{\partial x^2} = [f'(x+ay) + f'(x-ay).] --- (2)$$

from (1) and (2)

$$\frac{\partial^2 z}{\partial y^2} = a^2\frac{\partial^2 z}{\partial x^2}$$

b) $z = e^{k(r-x)}$, $r^2 = x^2 + y^2$

i). $z = e^{k\left(\sqrt{x^2+y^2}-x\right)}$

$$\frac{\partial z}{\partial x} = ke^{k\left(\sqrt{x^2+y^2}-x\right)}\left\{\frac{1}{2}(x^2+y^2)^{\frac{1}{2}}.2x - 1\right\} = kz\left\{\frac{x-\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\right\}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = k^2z^2\left\{\frac{x^2 - 2x\sqrt{x^2+y^2} + x^2 + y^2}{x^2+y^2}\right\}$$

$$= \frac{k^2z^2}{x^2+y^2}\left\{2x^2 + y^2 - 2x\sqrt{x^2+y^2}\right\} --- (1)$$

$$\frac{\partial z}{\partial y} = ke^{k\left(\sqrt{x^2+y^2}-x\right)}\left\{\frac{1}{2}(x^2+y^2)^{\frac{1}{2}}.2y\right\} = \frac{kzy}{\sqrt{x^2+y^2}}$$

$$\left(\frac{\partial z}{\partial y}\right)^2 = \frac{k^2z^2y^2}{x^2+y^2} --- (2)$$

$$2zk\frac{\partial z}{\partial x} = 2k^2z^2\left\{\frac{x-\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\right\}$$

$$= 2k^2z^2\left\{\frac{x\sqrt{x^2+y^2} - (x^2+y^2)}{x^2+y^2}\right\} --- (3)$$

(1) + (2) + (3)

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zk\ \frac{\partial z}{\partial x}$$

$$= \frac{k^2z^2}{x^2+y^2}\left\{2x^2 + y^2 - 2x\sqrt{x^2+y^2} + y^2 + 2x\sqrt{x^2+y^2} - 2x^2 - 2y^2\right\} = 0$$

ii). $z = e^{k\left(\sqrt{x^2+y^2}-x\right)}$

$$\frac{\partial z}{\partial x} = ke^{k\left(\sqrt{x^2+y^2}-x\right)}\left\{\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}.2x - 1\right\} = kz\left\{\frac{x-\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\right\} --(1)$$

$$\frac{\partial^2 z}{\partial x^2} = kz\left\{\frac{\sqrt{x^2+y^2}\times 1 - x\times\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\times 2x}{x^2+y^2}\right\}$$

$$+k^2z\left\{\frac{x}{\sqrt{x^2+y^2}} - 1\right\}\left\{\frac{x}{\sqrt{x^2+y^2}} - 1\right\}$$

$$\frac{\partial^2 z}{\partial x^2} = kz\left\{\frac{(x^2+y^2) - x^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2z\left\{\frac{x^2 - 2x\sqrt{x^2+y^2} + (x^2+y^2)}{x^2+y^2}\right\}$$

$$\frac{\partial^2 z}{\partial x^2} = kz\left\{\frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{2x^2+y^2-2x\sqrt{x^2+y^2}}{x^2+y^2}\right\} - - - (2)$$

$$\frac{\partial z}{\partial y} = ke^{k\left(\sqrt{x^2+y^2}-x\right)}\left\{\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}.2y\right\} = \frac{kzy}{\sqrt{x^2+y^2}}$$

$$\frac{\partial^2 z}{\partial y^2} = kz\left\{\frac{\sqrt{x^2+y^2}\times 1 - y\times\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\times 2y}{x^2+y^2}\right\}$$

$$+k^2 z\left\{\frac{y}{\sqrt{x^2+y^2}}\right\}\left\{\frac{y}{\sqrt{x^2+y^2}}\right\}$$

$$\frac{\partial^2 z}{\partial y^2} = kz\left\{\frac{(x^2+y^2)-y^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{y^2}{x^2+y^2}\right\}$$

$$\frac{\partial^2 z}{\partial y^2} = kz\left\{\frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{y^2}{x^2+y^2}\right\} - - - (3)$$

$(2) + (3) + 2k \times (1)$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2k\ \frac{\partial z}{\partial x} = kz\left\{\frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{2x^2+y^2-2x\sqrt{x^2+y^2}}{x^2+y^2}\right\}$$

$$+kz\left\{\frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{y^2}{x^2+y^2}\right\} + 2k\times kz\left\{\frac{x-\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\right\}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2k\ \frac{\partial z}{\partial x}$$

$$= kz\left\{\frac{x^2+y^2}{(x^2+y^2)^{\frac{3}{2}}}\right\} + k^2 z\left\{\frac{2x^2+y^2-2x\sqrt{x^2+y^2}+y^2}{x^2+y^2}\right\}$$

$$+2k^2 z\left\{\frac{x\sqrt{x^2+y^2}-(x^2+y^2)}{(x^2+y^2)}\right\}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2k\ \frac{\partial z}{\partial x} = \frac{kz}{r} + 2k^2 z\left\{\frac{(x^2+y^2)-x\sqrt{x^2+y^2}}{x^2+y^2}\right\}$$

$$-2k^2 z\left\{\frac{(x^2+y^2)-x\sqrt{x^2+y^2}}{(x^2+y^2)}\right\}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2k\ \frac{\partial z}{\partial x} = \frac{kz}{r}$$

c) If $z = f(x-2y) + F(3x+y)$

$$\frac{\partial z}{\partial x} = f'(x-2y) + F'(3x+y) \times 3$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x-2y) + F''(3x+y) \times 9$$

$$\frac{\partial z}{\partial y} = f'(x-2y) \times (-2) + F'(3x+y)$$

$$\frac{\partial^2 z}{\partial y^2} = f''(x-2y) \times 4 + F''(3x+y)$$

$$\frac{\partial^2 z}{\partial x \partial x} = f'(x-2y) \times (-2) + F'(3x+y) \times 3$$

$$\frac{\partial^2 z}{\partial x^2} + a\frac{\partial^2 z}{\partial x \partial y} + b\frac{\partial^2 z}{\partial y^2}$$

$$= f''(x-2y) + F''(3x+y) \times 9 \ + a\{f'(x-2y) \times (-2) + F'(3x+y) \times 3\} + b\big(f''(x-2y) \times 4 + F''(3x+y)\big)$$

$$= f''(x-2y)\{1 - 2a + 4b\} + F''(3x+y)\{9 + 3a + b\} \ = 0$$

Since $f''(x-2y) \neq 0,\ \ F''(3x+y) \neq 0$

$1 - 2a + 4b = 0 - - - (1)$ and $9 + 3a + b = 0 - - - (2)$

Solving equations

$$a = \frac{-35}{14} = \frac{-5}{2}, \quad b = \frac{-21}{14} = \frac{-3}{2}$$

4. a) $z = f(x,y)$, $x = e^u \cos v$, $y = e^u \sin v$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} e^u \cos v + \frac{\partial z}{\partial y} e^u \sin v - - - (1)$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$\frac{\partial z}{\partial v} = -\frac{\partial z}{\partial x} e^u \sin v + \frac{\partial z}{\partial y} e^u \cos v - - - (2)$$

$(1)^2 + (2)^2$

$$\left[\frac{\partial z}{\partial u}\right]^2 + \left[\frac{\partial z}{\partial v}\right]^2 = \left(\frac{\partial z}{\partial x} e^u \cos v + \frac{\partial z}{\partial y} e^u \sin v\right)^2 + \left(-\frac{\partial z}{\partial x} e^u \sin v + \frac{\partial z}{\partial y} e^u \cos v\right)^2$$

$$= e^{2u}\left[\left(\frac{\partial z}{\partial x}\right)^2 \cos^2 v + 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\cos v \sin v + \left(\frac{\partial z}{\partial y}\right)^2 \sin^2 v\right]$$
$$+ e^{2u}\left[\left(\frac{\partial z}{\partial x}\right)^2 \sin^2 v - 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\sin v \cos v + \left(\frac{\partial z}{\partial y}\right)^2 \cos^2 v\right]$$

$$= e^{2u}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]$$

b).

$$f(x,y) = \sqrt{y^2 - x^2}\ \sin^{-1}\frac{x}{y} + \frac{x^2 - y^2}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(y^2 - x^2)^{-\frac{1}{2}}(-2x)\ \sin^{-1}\frac{x}{y} + (y^2 - x^2)^{\frac{1}{2}}.\frac{1}{\left(1 - \frac{x^2}{y^2}\right)^{\frac{1}{2}}} \times \frac{1}{y}$$
$$+ \frac{(x^2 + y^2)^{\frac{1}{2}} \times 2x - (x^2 - y^2) \times \frac{1}{2}(x^2 + y^2)^{\frac{1}{2}}.2x}{(x^2 + y^2)}$$

$$\frac{\partial f}{\partial x} = \frac{-x}{(y^2 - x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y} + 1 + \frac{2(x^2 + y^2)x - x(x^2 - y^2)}{(x^2 + y^2)^{\frac{3}{2}}}$$

$$= \frac{-x}{(y^2 - x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y} + 1 + \frac{x^3 + 3xy^2}{(x^2 + y^2)^{\frac{3}{2}}} - - - (1)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(y^2 - x^2)^{-\frac{1}{2}}(2y)\ \sin^{-1}\frac{x}{y} + (y^2 - x^2)^{\frac{1}{2}}.\frac{1}{\left(1 - \frac{x^2}{y^2}\right)^{\frac{1}{2}}} \times \frac{-x}{y^2}$$

$$+\frac{(x^2+y^2)^{\frac{1}{2}}\times -2y-(x^2-y^2)\times \frac{1}{2}(x^2+y^2)^{\frac{1}{2}}.2y}{(x^2+y^2)}$$

$$\frac{\partial f}{\partial y}=\frac{y}{(y^2-x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y}\ -\ \frac{x}{y}-\left[\frac{2(x^2+y^2)y+y(x^2-y^2)}{(x^2+y^2)^{\frac{3}{2}}}\right]$$

$$=\frac{y}{(y^2-x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y}-\ \frac{x}{y}-\frac{3x^2y+y^3}{(x^2+y^2)^{\frac{3}{2}}}---(2)$$

$(1)\times x+(2)\times y$

$$x\left[\frac{\partial f}{\partial x}\right]+y\left[\frac{\partial f}{\partial y}\right]=\frac{-x^2}{(y^2-x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y}+x+\frac{x^4+3x^2y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

$$+\frac{y^2}{(y^2-x^2)^{\frac{1}{2}}}\ \sin^{-1}\frac{x}{y}-\ x-\frac{3x^2y^2+y^4}{(x^2+y^2)^{\frac{3}{2}}}$$

$$=(y^2-x^2)^{\frac{1}{2}}\ \sin^{-1}\frac{x}{y}\ +\ \frac{x^4-y^4}{(x^2+y^2)^{\frac{3}{2}}}$$

$$=(y^2-x^2)^{\frac{1}{2}}\ \sin^{-1}\frac{x}{y}\ +\ \frac{x^2-y^2}{(x^2+y^2)^{\frac{1}{2}}}$$

$$x\left[\frac{\partial f}{\partial x}\right]+y\left[\frac{\partial f}{\partial y}\right]=f(x,y)$$

5. a) $x+2y-3z=0---(1)$ $\qquad a_1x+b_1y+c_1z=d_1$

$2x-y-z=6---(2)$ $\qquad a_2x+b_2y+c_2z=d_2$

The direction ratios of the normal to the plane (1) is $1:2:-3$ and plane(2) is $2:-1:-1$

Hence the intersection line of the two planes has direction cosine

$b_1c_2-b_2c_1:c_1a_2-c_2a_1:a_1b_2-a_2b_1$

$2\times(-1)-(-1)(-3):(-3)\times 2-(-1)\times 1:1\times(-1)-2\times 2$

$-5 \quad : \quad -5 \quad : -5$

Let we can put $x=0$ $\qquad \therefore 2y-3z=0$

$$y+z=-6$$

$$5y=-18$$

$$y=\frac{-18}{5}$$

$$z=\frac{-12}{5}$$

Equation of the line

$$\frac{x}{1}=\frac{y+\frac{18}{5}}{1}=\frac{z+\frac{12}{5}}{1}$$

2nd Method

$$x+2y-3z=0 ---(1)$$
$$2x-y-z=6 \ ---(2)$$

$(1)-(2)\times 3 \qquad -5x+5y=-18$

$$x=y+\frac{18}{5}$$

$(1)+(2)\times 2 \qquad 5x-5z=12$

$$x=z+\frac{12}{5}$$

Equation of the line

$$\frac{x}{1}=\frac{y+\frac{18}{5}}{1}=\frac{z+\frac{12}{5}}{1}$$

Find the intersection point of the following line

$$\frac{x-1}{1}=\frac{y+1}{2}=\frac{z}{3} \qquad \frac{x}{1}=\frac{y+\frac{18}{5}}{1}=\frac{z+\frac{12}{5}}{1}$$

$$\frac{x-1}{1}=\frac{y+1}{2}=\frac{z}{3}=\lambda \qquad \frac{x}{1}=\frac{y+\frac{18}{5}}{1}=\frac{z+\frac{12}{5}}{1}=\mu$$

$$P\equiv\{\lambda+1,2\lambda-1,3\lambda\} \qquad Q\equiv\left\{\mu,\mu-\frac{18}{5},\mu-\frac{12}{5}\right\}$$

If P and Q are intersect

$$\lambda+1=\mu ---(A)$$
$$2\lambda-1=\mu-\frac{18}{5}---(B)$$
$$3\lambda=\mu-\frac{12}{5}---(C)$$

By (A) and (B)

$$\lambda=2-\frac{18}{5}=-\frac{8}{5}$$
$$\mu=1-\frac{8}{5}=\frac{3}{5}$$

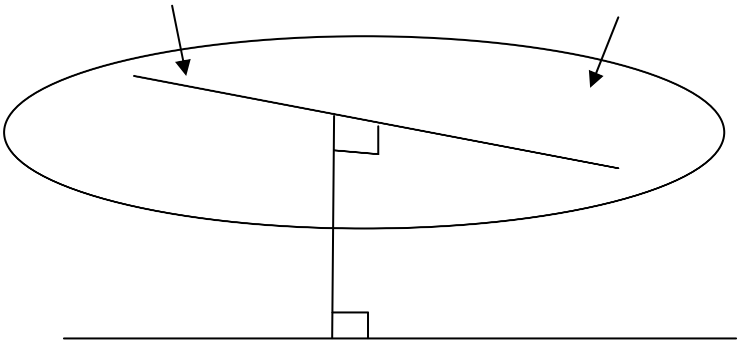
By (B) and (C)

$$\lambda=-\frac{12}{5}-\left(-\frac{18}{5}\right)=-\frac{6}{5}$$
$$\mu=3\times\left(-\frac{6}{5}\right)+\frac{12}{5}=\frac{6}{5}$$

λ and μ have different values

$\therefore$ These lines are not intersect

b)



Let assume that the equation of the required plane is $ax + by + cz = d$

The equation of m

$$\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+4}{5}$$

Since this line is parallel to the required plane

$\therefore$ The normal of the required plane P perpendicular to the line m

$\therefore \;\; 2a - 3b + 5c = 0 - - - (1)$

Since the line l passed through the required plane

$$l - \frac{x-1}{3} = \frac{y+6}{4} = \frac{z+1}{2}$$

$\therefore \;\; 3a + 4b + 2c = 0 - - - (2)$

$(1) \times 4 + (2) \times 3$

$17a + 26c = 0$

$$c = \frac{-11}{26}a$$

$3b = 2a + 5c$

$$= 2a - \frac{85}{26}a = \frac{-33}{26}a$$

$$b = -\frac{11}{26}a$$

$\therefore$ form of the required plane

$$ax - \frac{11a}{26}y - \frac{17a}{26}z = d$$

$$26x - 11y - 17z = d$$

Since this plane passed through the line l

$A \equiv (1, -6, -1)$lies on this plane

$$26 \times 1 - 11 \times (-6) - 17 \times (-1) = d$$

$\therefore d = 109$

$\therefore$ The equation of the required plane is $26x - 11y - 17z = 109 - - - (1)$

If this plane passes through the point $B \equiv (2,1,-4)$ it must be satisfy the equation (1)

$26 \times 2 - 11 \times 1 - 17 \times (-4) = 52 - 11 + 68 = 109$

$\therefore$ The point $(2,1,-4)$ lies on the above plane.

6. a) Find the intersection point of the following line

$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{3} = \lambda \qquad \frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2} = \mu$$

$$P \equiv \{\lambda + 1, \lambda + 1, 3\lambda + 2\} \qquad Q \equiv \{2\mu - 2, 5 - \mu, 2\mu - 3\}$$

If P and Q are intersect

$$\lambda + 1 = 2\mu - 3 \quad - - - (A)$$
$$\lambda + 1 = 5 - \mu \quad - - - (B)$$
$$3\lambda + 2 = 2\mu - 3 \quad - - - (C)$$

By (A) and (B)

$$2\mu - 3 = 5 - \mu$$
$$3\mu = 8$$
$$\mu = \frac{8}{3}$$
$$\lambda = \frac{16}{3} - 4 = \frac{4}{3}$$

By (B) and (C)

$$5\lambda + 4 = 7$$
$$\lambda = \frac{3}{5}$$
$$\mu = 4 - \frac{3}{5} = \frac{17}{5}$$

λ and μ have different values

$\therefore$ These lines are not intersect

b) Find the intersection point of the following line

$$\frac{x-4}{3} = \frac{y-1}{2} = z - 3$$
$$x + y + 2z = 4 - - - (1)$$
$$3x - 2y - z = 3 - - - (2)$$

$(1) \times 2 + (2) \qquad 5x + 3z = 11$

$$x = \frac{11 - 3z}{5}$$

$$x = \frac{-3z+11}{5} = \frac{z-\frac{11}{3}}{-\frac{5}{3}}$$

$(1) + (2) \times 2 \quad 7x - 3y = 10$

$$x = \frac{10+3y}{7}$$

$$x = \frac{3y+10}{7} = \frac{y+\frac{10}{3}}{\frac{7}{3}}$$

$$\frac{x-4}{3} = \frac{y-1}{2} = z - 3 = \lambda --- (1)$$

$$\frac{x}{3} = \frac{y+\frac{10}{3}}{7} = \frac{z-\frac{11}{3}}{-5} = \mu \quad --- (2)$$

P is the point on the line $\frac{x-4}{3} = \frac{y-1}{2} = z-3$ and

Q is the point on the line of intersection of the plane $x+y+2z=4$ and $3x-2y-z=3$

$$P \equiv \{3\lambda+4, 2\lambda+1, \lambda+3\} \qquad Q \equiv \left\{3\mu, 7\mu-\frac{10}{3}, -5\mu+\frac{11}{3}\right\}$$

If the line (1) and (2) are intersect

$$3\lambda + 4 = 3\mu \qquad --- (A)$$

$$2\lambda + 1 = 7\mu - \frac{10}{3} \quad --- (B)$$

$$\lambda + 3 = -5\mu + \frac{11}{3} --- (C)$$

By (A) and (B)

$$\frac{3\lambda+4}{3} = \frac{2\lambda+\frac{13}{3}}{7}$$

$$21\lambda + 28 = 6\lambda + 13$$

$$\lambda = -1$$

$$\mu = \lambda + \frac{4}{3} = \frac{1}{3}$$

By (B) and (C)

$$7\mu - \frac{13}{3} = -10\mu + \frac{4}{3}$$

$$17\mu = \frac{17}{3}$$

$$\mu = \frac{1}{3}$$

$$\lambda = -5\mu + \frac{2}{3}$$

$$\lambda = -1$$

$\lambda = -1$ and $\mu = \frac{1}{3}$

λ and μ have same values

$\therefore$ These lines are intersect

$\therefore$ Equation of the plane which contains the two lines

$$\frac{x-4}{3} = \frac{y-1}{2} = z - 3 = -1$$

Let assume that the equation of the plane is $ax + by + cz = d$

Since the lines $\frac{x-4}{3} = \frac{y-1}{2} = z - 3$ and $\frac{x}{3} = \frac{y+\frac{10}{3}}{2} = \frac{z-\frac{11}{3}}{-5}$

$$\therefore 3a + 2b + c = 0 --- (1)$$

$$3a + 7b - 5c = 0 --- (2)$$

$(2) - (1)$

$$5b - 4c = 0$$

$$b = \frac{4c}{5}$$

$$3a + \frac{8c}{5} + c = 0 \qquad a = \frac{-13c}{15}$$

$$\frac{-13c}{15}x + \frac{4c}{5}y + cz = d$$

$$-13x + 12y + 15z = 15d$$

$P \equiv (1, -1, 2)$ lies this plane

$$-13 - 12 + 30 = 15d$$

$$d = \frac{29}{15}$$

$$-13x + 12y + 15z = -29$$